A fully abstract semantics for concurrent graph reduction

A A EFFR EY

AB AC — s p p r pr s nts u str t s nt s or v r nt t unt p λ u us t r urs v r t ons — rst pr s nt su r st n or on u str t on or t un t p λ u us on ntr t n on AB A Y n G s or on t λ u us AB A Y n G s or s s on t ost out r ost r u t on t out s r n — s s not n nt n n p nt t ons o s r n r u n s nt

1 Introduction

s p p r s out t r t ons p t n t o s o o put r s n *full abstraction* n *concurrent graph reduction* Fu str t on s t stu o r t n not t on n op r t on s nt s Con urr nt r p r u t on s n nt p r p nt t on t n qu or non str t un t on pro r n n u s

nt s p p r pp t t n qu s o AB A Y n G to pr s nt u str t not t on s nt s or t on urr nt r p r u t on or t v n n EY E s t t oo

n o n so us t o s ro u str t on o p r p nt t on n on urr n t or

1.1 Full abstraction

Fu str t on or n n B p or s t r t ons p

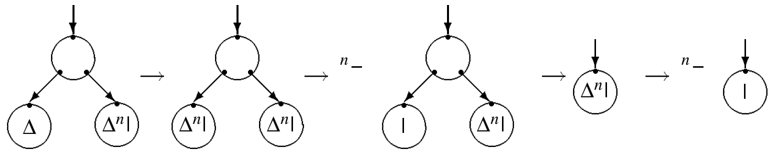
t s v op AD H s n p nt t on t ost out r ostr u t on H o s r t t t ost out r ostr u t on nt pon nt t to u t n pr ss on u to oss *sharing* nor t on For p n

$$\mathsf{I} = \lambda x\,.\,x \qquad \Delta = \lambda x\,.\,xx \qquad M\ N = N \qquad M^{n+}\ N = M(M^n N)$$

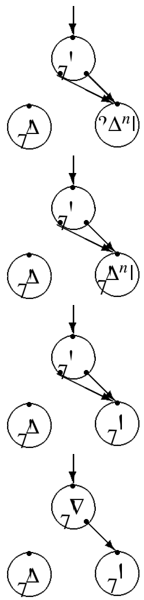
nt v u t on $\Delta^{n+}\ \mathsf{I} \rightarrow^{*} \mathsf{I}$ s

$$\Delta^{n+}\ \mathsf{I} \rightarrow (\Delta^n\,\mathsf{I})(\Delta^n\,\mathsf{I}) \rightarrow\ ^{n-}\ \mathsf{I}(\Delta^n\,\mathsf{I}) \rightarrow \Delta^n\,\mathsf{I} \rightarrow\ ^{n-}\ \mathsf{I}$$

us $\Delta^n\mathsf{I}$ t s $^{n}-$ r u tons to t r n t $-$ s pon nt o up s us op n $\Delta^n\mathsf{I}$ n t r u t on $\Delta^{n+}\ \mathsf{I} \rightarrow (\Delta^n\,\mathsf{I})(\Delta^n\,\mathsf{I})$ n n r s n r t s nt tr s or t s r u t on r not s un t on pp t on



s n n s us t p nt t on β r u t on t su st tu t on $-$ n r u $(\lambda w\,.\,M)N \rightarrow M[N/w]$ s p r t op N or o urr n w n M n op t n s to r u s p r t $-$ n r o t s n n r t r t n op n t r s op *pointers to* t r s t t s r u s nt *graphs* r t r t n s nt *trees* For p t



→ up t n

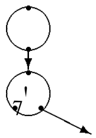
→ p n tr v rs

→ ^{n} n u t on

→ up t n

→ 7'

not *con uent* or *Church Rosser*, s n sp n tr v rs



- G r o t on s s nt un port nt so r p n on r t n on r t out r o t n — n ou p t t s to tru s n r o t on s ntro u on us or t t ons —
- n s s nt un port nt so r p n on r rr sp t t r ts no s r t or not — n p rt u t s ns t t on urr nt u t on s s nt qu nt to s qu nt u t on —
- r nt tr nsp r n ns t t t s s nt un port nt r p ont ns op no or po nt r to no —

r r nu r pp t ons or u str t s nt s

B FY G C B ZA — A nu r o p rs non str t un t on n u s not H s o p r or t *G* n us opt t ons — n opt rs not *peephole optimizers* EY E C — r p on s t r t not r s nt qu nt ut or nt t r — s nt s s orr t t n no t t n su opt t on t s op r t on our n ont ts —

nortun t t s nt s s not o p t n t r op t t ons t t r not s nt qu nt n t r s t pt t on or t o p r r t r to us *ad hoc* r son n to ust s nt n opt t on ont roun s t t t s nt s s too n — t s nt s s u

2 Tree reduction

s C pt r pr s nts su r st n or on u str t o s or t ost out r ost r u t on t unt p λ u us t on ntr t s on AB A Y n G s or on t λ u us ut so n u s t r ro AB A Y BA E D EG BA A D EG *et al.* B D E CE n

2.1 The λ-calculus with P

n t s C pt r s uss t t or v op AB A Y n G s on *leftmost outermost* r u t on s s t s nt s s o t *non strict* un t on n u ssu s A G s FA B s on r E s G r E s r n n H s H DA *et al.*

n t unt p λ u us pr ss ons r un t ons n t s un t ons t un t ons s nputs n r turn ot r un t ons n r r t s s pur t or o put t on str t ro ons r t ons t unt p λ u us s t r or s pr ss on

- A *free variable x*
- An *application MN*
- An *abstraction* $\lambda x\,.\,M$

u t r s r *sequential* n t on or o put t on s β r u t on r n str t on s pp $(\lambda x\,.\,M)N \to M[N/x]$ Fo o n ou p t t t n n u str t s nt s u s p r so or p r o put t on r r nu r poss p r o n tors on n us p r on t on AB A Y n G us p r on v r n n B D s n

ours v s to ω *continuous* un t ons t t s

$$a \; s \; t \quad t \; a \le a \le \cdots$$

t n

$$fa \; s \; t \quad t \; fa \le fa \le \cdots$$

For p t s rs t odd un t on s n

$$- \; s \; t \quad t \quad \le - \le - \le \cdots$$

ut

$$- \; s \; not \; t \quad t \; - \le - \le — \le \cdots$$

n t not t on s nt s Λ_P n $\mathbf{D} \simeq (\mathbf{D} \to \mathbf{D})_\perp$ o s o t t su **D** ust st pr s nt t s t t s qu n n t o ns **D** , **D** , . . . r

$$\mathbf{D} = \qquad \mathbf{D}_{n+} = (\mathbf{D}_n \to \mathbf{D}_n)_\perp$$

s n so pr s nt s t po nt *functor* F t n o ns

$$F\mathbf{D}_i = (\mathbf{D}_i \to \mathbf{D}_i)_\perp = \mathbf{D}_{i+}$$

n n or r to s o t t **D** sts s o t t F s ont nuous n or r to o t s pr s nt

- A not on *domain* su t t t on po nt o n s o n n F s un tor t n o ns
- A not on *order* t n o ns t st nt n r v r n o ns s t
- A not on *continuous functor* t n o ns su t t F s ont nuous

Fo o n us t *category of* ω *cpo s with embeddings* s t pprop r t not on or r o ns n F s ont nuous un tor t ust v st po nt us s our n t on **D**
r st t s s t on pr s nt t t n t s t s onstru t on
s n t s ort r n r so s p t or t or nt r st

EXA E lift $\mathrm{C} \to \mathrm{C}_\perp$ s un tor s n v

- n o t lift A in $\mathrm{C}_\perp$ or A A rro e^R s un qu n so e $A \to B$ in CPOE n f $B \to A$ in ωCPOE t n

$$(e \circ f \leq \mathrm{id}, f \circ e = \mathrm{id}) \quad \text{p} \quad s\, e^R = f$$

$()_\perp$ ωCPOE → ωCPOE s t t n un tor t

- $A_\perp$ in ωCPOE or A in ωCPOE
- $e_\perp$ $A_\perp \to B_\perp$ in ωCPOE or e $A \to B$ in ωCPOE

Δ ωCPOE → ωCPOE s t on un tor t

- $\Delta A = (A, A)$ in ωCPOE or A in ωCPOE
- $\Delta f = (f, f)$ $\Delta A \to \Delta B$ in ωCPOE or f $A \to B$ in ωCPOE

$(\to)$ ωCPOE → ωCPOE s t ω ont nuous un t on sp un tor t

- $(A \to B)$ in ωCPOE or (A, B) in ωCPOE
- $(e \to f)$ $(A \to B) \to (A' \to B')$ in ωCPOE or (e, f) $(A, B) \to (A', B')$ in ωCPOE

r $e \to f$ s n

$$(e \to f) g = f \circ g \circ e^R$$
$$(e \to f)^R g = e \circ g \circ f^R$$

s t n t o t n ωCPOE □

DEF A o on $\{e_i \;\; A_i \to A \text{ in } \omega\text{CPOE} \mid i \text{ in } \omega\}$ s *determined* $\bigvee\{e_i \circ e_i^R \mid i \text{ in } \omega\} = \mathrm{id}$ □

Any determined cocone is a colimit.

F t $\{e_i \;\; A_i \to A \mid i \text{ in } \omega\}$ t r n o on n ω n $\{e_i^j \;\; A_i \to A_j \mid i \leq j \text{ in } \omega\}$ n or n ot r o on $\{f_i \;\; A_i \to B \mid i \text{ in } \omega\}$ n g $A \to B$ s

$$g = \bigvee\{f_i \circ e_i^R \mid i \text{ in } \omega\}$$
$$g^R = \bigvee\{e_i \circ f_i^R \mid i \text{ in } \omega\}$$

n n s o t t g s t un qu n su t t $g \circ e_i = f_i$ us $\{e_i \;\; A_i \to A \mid i \text{ in } \omega\}$ s o t □

Any ω chain in ωCPOE has a determined cocone.

F t $\{e_i^j \;\; A_i \to A_j \mid i \leq j\}$ n ω n An *instantiation* t s n s un t on f su t t

$$\mathrm{dom}\, f = \omega \qquad fi \in A_i \qquad e_i^{jR}(fj) = fi$$

t n n

$$A = \{f \mid f \text{ s n nst nt t on}\}$$

t t po nt s or r n s s n ω po t on

$$\bigvee\{f_i \mid i \text{ in } \omega\} j = \bigvee\{f_i j \mid i \text{ in } \omega\}$$

n n

$$e_i a j = \begin{cases} e_i^j a & i \leq j \\ e_j^{iR} a & \text{ot r s} \end{cases}$$

$$e_i^R f = fi$$

n s o t t $\{e_i \;\; A_i \to A \mid i \text{ in } \omega\}$ s t r n o on □

DEF D s t t r n o t t ω n

$$\mathbf{D} =$$
$$\mathbf{D}_{i+} = (\mathbf{D}_i \to \mathbf{D}_i)_\perp$$

t e_i $\mathbf{D}_i \to \mathbf{D}$ in ωCPOE n ropos t on n **D** s t n t po nt t un tor $()_\perp \circ (\to) \circ \Delta$ n ropos t on □

2.6 Logical presentation of D

n t on n str t pr s nt t on **D** us n t t or ω po s t n nt s s t on pro on r t pr s nt t on **D** s r to C s *information systems* Fo o n AB A Y s *domain theory in logical form* us t pro o Φ s n t rn t pr s nt t on **D** n p rt u s o t tt ω po *lters* Φ s qu nt to **D**

DEF Ψ ⊆ Φ s *lter*

- ω ∈ Ψ
- φ ∈ Ψ n ⊢ φ ≤ ψ t n ψ ∈ Ψ
- φ, ψ

- $\vdash \phi \leq \psi \iff [\![\phi]\!] \leq [\![\psi]\!]^{-}$
- a s ω

Fo o s ro t n t on pt

a = ⊥ t n

$$a = \bot = \bigvee \emptyset = \bigvee\{b \mapsto c \mid b \mapsto c \le a\}$$

t r s n s o t t or n d

$$\mathsf{apply}\, a d = \mathsf{apply}(\bigvee\{b \mapsto c \mid b \mapsto c \le a\}) d$$

n so $a = \bigvee\{b \mapsto c \mid b \mapsto c \le a\}$

$a \mapsto b \le \bigvee C$ or n n $C \subseteq \mathbf{D}$ t n

$$b = \mathsf{apply}(a \mapsto b) a \le \mathsf{apply}(\bigvee C) a = \bigvee\{\mathsf{apply}\, c a \mid c \in C\}$$

n b s o p tt r s $c \in C$ su t t $b \le \mathsf{apply}\, c a$ so

$$a \mapsto b \le a \mapsto \mathsf{apply}\, c a \le c$$

us $a \mapsto b$ s o p t

$a \mapsto b \le \bigvee A$ or n t s t $A \subseteq \mathbf{D}$ t n

$$b = \mathsf{apply}(a \mapsto b) a \le \mathsf{apply}(\bigvee A) a = \bigvee\{\mathsf{apply}\, c a \mid c \in A\}$$

n b s pr t r s $c \in A$ su t t b

$\Rightarrow \nu x\,.\,\Gamma \vdash \lambda x\,.\,M \quad \psi_i \rightarrow \chi_i$ (→I)

$\Rightarrow \Gamma \vdash \lambda x\,.\,M \quad \psi_i \rightarrow \chi_i$ (≤)

us (∧I) n (≤), $\Gamma \vdash \lambda x\,.\,M \quad \phi^-$ □

s st to t rt not t on n proo t or t pr s nt t ons o t o n n st rt to n t s t t op r t on pr s nt t on ¯ o n t s o t tt not t on s nt s r sp ts t op r t on s nt s o o n BA E D EG s n t on o λ *theory*

$(M \sqsubseteq_D N \Rightarrow M \sqsubseteq_S N)$ For n Γ n ϕ, $M \sqsubseteq_D N$ t n

$$
\begin{aligned}
& \Gamma \vdash M \quad \phi \\
& \quad \Rightarrow [\![\phi]\!] \leq [\![M]\!][\![\Gamma]\!] && \text{ropn} \\
& \quad \Rightarrow [\![\phi]\!] \leq [\![N]\!][\![\Gamma]\!] && \text{H pot s s} \\
& \quad \Rightarrow \Gamma \vdash M \quad \phi && \text{ropn}
\end{aligned}
$$

us $M \sqsubseteq_D N$ t n $M \sqsubseteq_S N$ −

$(M \sqsubseteq_S N \Rightarrow M \sqsubseteq_D N)$ For n σ, $M \sqsubseteq_S N$ t n

$$
\begin{aligned}
& [\![M]\!]\sigma \\
& \quad = \bigvee\{[\![\phi]\!] \mid [\![
\end{aligned}
$$

- rec D in M s *recursive declaration* ❧ D n M −

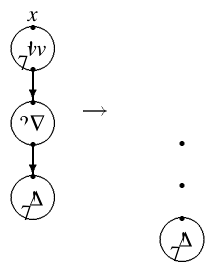
EXA E –

- $x = 7M$,

pp t on o *M* to ts t s r n n r n

$$x = u \quad v,$$
$$u = \nabla z,$$
$$v = \nabla z,$$
$$z = M$$

ζ p



DEF ↦ s v n o s

(BUILD) $x = (\mathsf{rec}\, D \,\mathsf{in}\, M) \mapsto \mathsf{local}\, D \,\mathsf{in}\, (x = M)$

(∇TRAV) $x = \nabla y, y = ?M \mapsto x = \nabla y, y = M$

(TRAV) $x = y \; z, y = ?M \mapsto x = y \; z, y = M$

(∨TRAV) $x = y \vee z, y = ?M \mapsto x = y \vee z, y = M$

(∇UPD) $x = \nabla y, y = \lambda w.\, M \mapsto x = \lambda w.\, M, y = \lambda w.\, M$

(UPD) $x = y \; z, y = \lambda w.\, M \mapsto x = M[z/w], y = \lambda w.\, M$

(∨UPD) $x = y \vee z, y = \lambda w.\, M \mapsto x = \mathsf{l}, y = \lambda w.\, M$

(γ) $\nu(\mathsf{wv}\, D)\,.\, D \mapsto \varepsilon$

n stru tur ru s

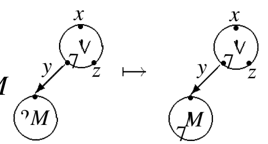
$$(\mathrm{L})\ \frac{D \mapsto E}{D, F \mapsto E, F} \qquad (\mathrm{R})\ \frac{D \mapsto E}{F, D \mapsto F, E} \qquad (\nu)\ \frac{D \mapsto E}{\nu x\,.\, D \mapsto \nu x\,.\, E}$$

ot t t $D \mapsto E$ t n $\mathsf{rv}\, D \supseteq \mathsf{rv}\, E$ n $\mathsf{wv}\, D = \mathsf{wv}\, E$ −

- $D \to E$ $D \equiv\mapsto\equiv E$ −
- $D \to E$ $D \equiv E$ n $D \to^{n+} E$ $D \to\to^{n} E$ −
- $D \to^{*} E$ $\exists n\,.\, D \to^{n} E$ −
- $D \to^{\leq i} E$ $\exists n \leq i\,.\, D \to^{n} E$ − □

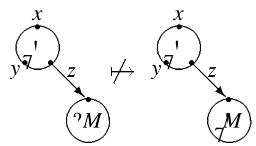
EXA E

$x = \neg y \vee z, y = ?M$
$\mapsto x = \neg y \vee z, y = \neg M$

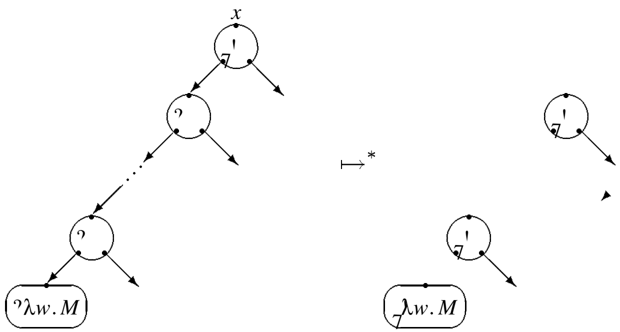


ot t t s n r o n v u t on v

$x = \neg y \; z, z = ?M$
$\not\mapsto x = \neg y \; z, z = \neg M$

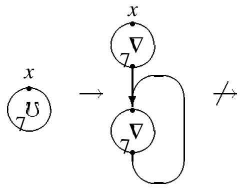


s p s s sp n tr v rs us t r t n sp n un t n r t on pp t on n or no s t For p



Ho v r t o or r o t on n v o v s r p s o r tr r s
n so s u r r r nu r t n so ss s op or on urr n ¯n p

n r n r p s



$$\mathsf{set}\,X\,f\,g\,\sigma\,x = \begin{cases} f(g\sigma)x & x \in X \\ \sigma x & \text{ot r s} \end{cases}$$

$$\mathsf{fix}\,f = \bigvee\{f^n\bot \mid n \;\mathsf{in}\;\omega\}$$

- $M \sqsubseteq_D N$ $[\![M]\!] \leq [\![N]\!]$ –
- $D \sqsubseteq_D E$ $\mathsf{wv}\,D = \mathsf{wv}\,E$ n $[\![D]\!] \leq [\![E]\!]$ – □

EXA E – n s o t t t s nt s t o r t r s ⊤ s n

$[\![\mathsf{rec}\,x = \lambda y.\,\nabla x \;\mathsf{in}\; \nabla x]\!]$

$= [\![\nabla x$

7

ys ts s ψ t n

$$w =_7 \lambda y.\, w, x =_7 \lambda y.\, w, z =_7 x \quad y, D$$
$$\to w =_7 \lambda y.\, w, x =_7 \lambda y.\, w, z =_7 \nabla w, D$$
$$\to w =_7 \lambda y.\, w, x =_7 \lambda y.\, w, z =_7 \lambda y.\, w, D$$

n u t ons s t s s z χ — us

$$(w =_7 \lambda y.\, w, x =_7 \lambda y.\, w) \quad \phi$$

Fro t s t ss p to s o t t $(w =_7 \lambda y.\, w)$ $(w \quad \phi)$ —

s n t on p n s on t not on r p t ns on st pr or r $D \sqsubseteq E$ —

DEF $D \sqsubseteq E$ n n $\vec{x}$ $\vec{y}$ D' n E' su t t

$$D \equiv \nu \vec{x}.\, D' \qquad E \equiv \nu \vec{x}\vec{y}.\, (D', E') \qquad \mathrm{fv}\, D \cap \vec{y} = \emptyset$$

ot t t $\sqsubseteq$ s pr or r n t t $D \sqsubseteq E \sqsubseteq D$ $D \equiv E$ — □

n t n n t t op r t on nt rpr t t on t o —

DEF For os r t ons $\models D \quad \Delta$ s v n t o s

$$(\varepsilon\mathrm{I}) \quad \models D \quad \varepsilon \qquad (\omega\mathrm{I}) \quad \models D \quad (x \quad \omega)$$

n stru tur ru s

$$(\wedge\mathrm{I})\ \frac{\models D \quad \Gamma \qquad \models D \quad \Delta}{\models D \quad \Gamma \wedge \Delta} \qquad (\to\mathrm{I})\ \frac{D{\Downarrow_x} \qquad \forall (z =_7 x \quad y) \sqsubseteq E \sqsupseteq D.\ \models E \quad (y \quad \phi) \Rightarrow \models E \quad (z \quad \psi)}{\models D \quad (x \quad \phi \to \psi)}$$

s n n r to n D n n $\Gamma \models D \quad \Delta$

$$\forall E.\, (\models D, E \quad \nu(\mathrm{wv}\, D).\, \Gamma) \quad \text{p} \quad \text{s}\, (\models D, E \quad \Delta)$$

r $\Gamma \models M \quad \phi$

$$\forall D, z.\, (\models (D, z =_7 M) \quad \Gamma) \quad \text{p} \quad \text{s}\, (\models (D, z =_7 M) \quad (z \quad \phi))$$

n ons qu n u str t on s t t or λ u us t r s t s op r t on n t on r s t t n t on t on — □

n n pro s st or Lam s or Λ_P — s us s t s propo s t ons n v u nts t or $\Gamma \vdash M \quad \phi$ n $\Gamma \vdash D \quad \Delta$ — n r n t nt pro s st or Lam n t t Λ_P s t pro s st or r urs v r t ons — ot t t

- pro ru s $(_7)$ n $(^?)$ or t n unt r t ons r t s — nt t r s no r n t n t or n unt no on u t s s n $\phi = \psi \rightarrow \chi$

Ξ

n n

(7)

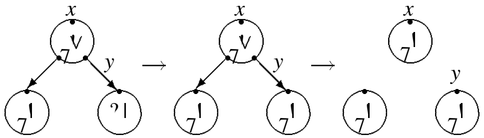
$$\partial[\![D,E]\!] = (X \cup X', Y \cup Y', Z \cup Z', f \cup f')$$

$$\partial[\![\nu x\,.\,D]\!] = (X \setminus \{x\}, Y \cup \{x\}, Z, f)$$

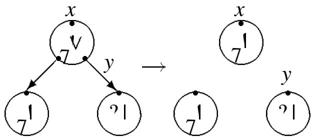
r $\partial[\![D]\!] = (X, Y, Z, f)$, $\partial[\![E]\!] = (X', Y', Z', f')$ n X, Y, X' n Y' r s o nt − □

n n s o t t t s s nt s s u str t or ≡ −

For p o t r u t on



ut not



ot t t n t r o stor p t o

$$(x = y \vee z, y = \lambda w. M) \mapsto (x = \,, y = \lambda w. M)$$

s n to ons r t s s n $x = z$ $y = z$ n $x \neq z \neq y$

DEF $\mapsto_c$ s n o s

(BUILD)	$x = (\mathsf{rec}\, D \,\mathsf{in}\, M) \mapsto_c \mathsf{rec}\, D \,\mathsf{in}\, (x = M)$
(∇TRAV)	$x = \nabla y, y = M \mapsto_c x = \nabla y, y = M$
(TRAV)	$x = y \quad z, y = M \mapsto_c x = y \quad z, y = M$
($\vee$TRAV)	$x = y \vee z, y = M \mapsto_c x = y \vee z, y = M$
(∇UPD)	$x = \nabla y, y = \lambda w. M \mapsto_c x = \lambda w. M, y = \lambda w. M$
(UPD)	$x = y \quad z, y = \lambda w. M \mapsto_c x = M[z/w], y = \lambda w. M$
($\vee$UPDa)	$x = y \vee z, y = \lambda w. M, z = N \mapsto_c x = \,, y = \lambda w. M, z = N$
($\vee$UPDb)	$x = y \vee y, y = \lambda w. M \mapsto_c x = \,, y = \lambda w. M$
($\vee$UPDc)	$x = y \vee x, y = \lambda w. M \mapsto_c x = \,, y = \lambda w. M$

n stru tur ru s

(L) $\dfrac{D \mapsto_c E}{D, F \mapsto_c E, F}$ (R) D

For closed D

$\leq$ is a partial order.
$D \leq \equiv E$ iff $D \equiv \leq E$.
If $D \to_\gamma \to_c E$ then $D \to_c \to_\gamma E$.
If $D \leq \to_c E$ then $D \to_c^{\leq} \leq E$.
If $D \leq \to_\gamma E$ then $D \to_\gamma \leq E$.
If $D \to E$ then $D \to_c^* \to_\gamma^* \leq E$.
If $D \to^* E$ then $D \to_c^* \to_\gamma^* \leq E$.

F

B nt on $\leq$ sr v B n u t on on t proo $D \leq E$ n s o t t $D \leq E \leq D$ t n $D =$

$$\begin{array}{lll}
\rightarrow_c & \nu\vec{x}\vec{y}\,.\,(F', z = M, G) & \vee\text{TRAV} \\
\rightarrow_c & \nu\vec{x}\vec{y}\,.\,(F', z = M, H) & \vee\text{UPD}a \\
\equiv & \nu\vec{x}\,.\,(\nu\vec{y}\,.\,(F', z = M), H) & \nu\text{MIG} \\
\leq & \nu\vec{x}\,.\,(\nu\vec{y}\,.\,(F', z = M), H) & \text{D n } \leq \\
\equiv & \nu\vec{x}\,.\,(F, H) & \text{Eqn} \\
\equiv & E & \text{Eqn}
\end{array}$$

us $D \rightarrow_c^* \rightarrow_\gamma^* \leq E$ –

(OTHERS) ot r o s r o s $\mapsto$ n so $D \rightarrow_c^* \rightarrow_\gamma^* \leq E$ –

– t $D \rightarrow^n E$ n pro n u t on on n

(

– If $D \equiv (D', x = \lambda w.M) \to_c E$ then $E' \equiv (E', x = \lambda w.M)$

– If $D \equiv (D', x = M) \to_c E$ then $E \equiv (D', x = M)$

or $E \equiv (E', x =$

n ropos t on – t r

- v

$$H \equiv (G, \mathsf{local}\ K\ \mathsf{in}\ x = M)$$

n s

$$\begin{aligned} E & \\ &\equiv \nu\vec{x}\,.\,(G, J) && \text{Eqn} \\ &\equiv \nu\vec{x}\,.\,(G, \mathsf{local}\ K\ \mathsf{in}\ x = M) && \text{Eqn} \\ &\equiv \nu\vec{x}\,.\,H && \text{Eqn} \\ &\equiv F && \text{Eqn} \end{aligned}$$

- or v

$$H \equiv (L, x = \mathsf{rec}\ K\ \mathsf{in}\ M$$

n or n N

$$(G, x$$

r u t on s n n or r to $\forall$ u t x

–If $D \vdash x \prec y$ then $D, E \vdash x \prec y$.
–If $\nu x . D \vdash y \prec z$ then $D \vdash y \prec z$.
–If $x \neq y \neq z$ w is fresh and $D \vdash x \prec z$ then $[w/y]D[w/y] \vdash x \prec z$.

F – n u t ons on t pro ≺ –

n n s o t t $D \to_x E$ t r s r u t on on t x sp n D

– $D \to_x E$ iff $D \equiv \nu\vec{x} . F$ $E \equiv \nu\vec{x} . G$ $F \to_y G$ is an axiom and $F \vdash x \prec y$.

F –

⇒ An n u t on on t pro $D \to_x E$ –
⇐ An n u t on on t pro $F \vdash x \prec y$ –

– If $D \vdash x \prec y$ and $D \to_y E$ then $D \to_x E$.

F B ropos t on $D \equiv \nu\vec{x} . F$, $E \equiv \nu\vec{x} . G$, $F \vdash y \prec z$ n $F \to_z G$ s n o – n ropos t on $F \vdash x \prec y$ so (TRANS) $F \vdash x \prec z$ so ropos t on $D \to_x E$ –

–

–If $D \equiv (D', D'')$ $D \vdash x \prec z$ $x \in$ wv D' and $z \in$ wv D''
then $\exists y \in$ rv $D' \cap$ wv D'' . $D' \vdash x \prec y$.
–If $D \equiv (D', D'')$ $D \vdash x \prec z$ and $x, z \in$ wv D' then $D' \vdash x \prec z$
or $\exists y \in$ rv $D' \cap$ wv D'' . $D' \vdash x \prec y$.

F –An n u t on on t pro $D \vdash x \prec z$ –

– on u t s s r (ν) n (TRANS) –n t s (ν) v

$$D = \nu w . E \qquad E \vdash x \prec z \qquad x \neq w \neq z$$

n ropos t on – t r

- $D' \equiv \nu w . E'$ n $E \equiv (E', D'')$ so n u t on n $y \in$ rv $E' \cap$ wv D'' su t t $E' \vdash x \prec y$ – n $y \in$ wv D'' so $y \neq w$ so $y \in$ rv $D' \cap$ wv D'' n (ν) $D' \vdash x \prec y$ –
- $D'' \equiv \nu w . E''$ n $E \equiv (D', E'')$ so n u t on n $y \in$ rv $D' \cap$ wv E'' su t t $D' \vdash x \prec y$ – n $y \in$ rv D' so $y \neq w$ so $y \in$ rv $D' \cap$ wv D'' –

n t s (TRANS) v

$$D \vdash x \prec w \prec z$$

n t r

- $w \in$ wv D' so n u t on on rt t r
  - $D' \vdash x \prec w$ n n u t on D n $y \in$ rv $D' \cap$ wv D'' su t t $D' \vdash w \prec y$ so (TRANS) $D' \vdash x \prec y$ –
  - $\exists y \in$ rv $D' \cap$ wv D'' . $D' \vdash x \prec y$
- $w \in$ wv D'' so n u t on n $y \in$ rv $D' \cap$ wv D'' su t t $D' \vdash x \prec y$ –

ot r s s r s p r –

– s s r –

DEF –

- x s t n $x = M$ –
- x s unt n $x = ?M$ –
- x s unt n D, E x s unt n D or E –
- x s unt n $\nu y . D$ $x \neq y$ n x s unt n D –

–For closed D

–If $D \to_c (E', y = ?M) \equiv E$ then $D \equiv (D', y = ?M)$ and D' ⇒

n so

$$
\begin{aligned}
& D \\
& \quad \equiv \nu\vec{x}\,.\,(F, G) && \text{Eqn} \\
& \quad \equiv \nu\vec{x}\,.\,(F
\end{aligned}
$$

$\equiv \nu y\vec{w}.(G',H)$ (νMIG) n (νSWAP)

n

$$\begin{aligned} & \nu\vec{w}.(G',H) \\ & \rightarrow_c \nu\vec{w}.(G',I) && \text{Eqn} \\ & \equiv E' && \text{Eqn} \end{aligned}$$

- or v

$$I \equiv \nu y.I' \qquad E' \equiv \nu\vec{w}.(G,I')$$

so n s s o o t t ou v $H \mapsto_c I$ n t t t on poss t s (BUILD) n s

$$D \equiv \nu\vec{w}.(G,z = \mathsf{rec}\,F \;\mathsf{in}\; M)$$
$$E \equiv \nu\vec{w}.(G,\mathsf{local}\,F \;\mathsf{in}\; z = M)$$
$$E' \equiv \nu\vec{w}.(G,I')$$
$$\nu y.I' \equiv \mathsf{local}\,F \;\mathsf{in}\; z = M$$

B ropos t on

$$D \equiv \nu\vec{y}.(G,H) \qquad E \equiv \nu\vec{y}.(G,I) \qquad H \mapsto_c I \text{ s n o}$$

n ropos t ons n v

$$D' \equiv \nu\vec{y}.D''$$
$$(D'',x = M) \equiv (G,H)$$
$$E' \equiv \nu\vec{y}.E''$$
$$(E'',x = M) \equiv (G,I)$$

n ropos t ons n t r

- v

$$G \equiv (G',x = M) \qquad D'' \equiv (G',H) \qquad E'' \equiv (G',I)$$

so or n N

$$\begin{aligned} & D',x = N \\ \equiv\ & (\nu\vec{y}.D''),x = N && \text{Eqn} \\ \equiv\ & (\nu\vec{y}.(G',H)),x = N && \text{Eqn} \\ \mapsto_c\ & (\nu\vec{y}.(G',I)),x = N && \text{Eqn} \\ \equiv\ & (\nu\vec{y}.E''),x = N && \text{Eqn} \\ \equiv\ & E',x = N && \text{Eqn} \end{aligned}$$

- or v

$$H \equiv (H',x = M) \quad D'' \equiv (G,H') \quad I \equiv (I',x = M) \quad E'' \equiv (G,I')$$

n s n s s o o ou v $H \rightarrow_c I$ n t t t r

- $G,H \rightarrow_x G,I$ n so $D \rightarrow_x E$
- For n N, $H',x = N \rightarrow I',x = N$ n so $D',x = N \rightarrow E',x = N$

B ropos t on

$$D \equiv \nu\vec{x}.F \qquad E \equiv \nu\vec{x}.G \qquad F \vdash y \prec z \qquad F \rightarrow_z G \text{ s n o}$$

n n α on, rt so t t $x \notin x$ n ropos t on t r

- v

$$\vec{x} = \vec{y}w\vec{z} \qquad D' \equiv \nu\vec{y}\vec{z}.[x/w]F[x/w]$$

n so

$$\begin{aligned} & E \\ \equiv\ & \nu\vec{x}.G && \text{Eqn} \\ \equiv\ & \nu\vec{y}w\vec{z}.G && \text{Eqn} \\ \equiv\ & \nu x\vec{y}\vec{z}.[x/w]G[x/w] && (\alpha) \text{ n } (\nu\text{SWAP}) \end{aligned}$$

ropos t on

$$[x/w]F[x/w] \rightarrow_z [x/w]G[x/w]$$

ropos t on

$$[x/w]F[$$

- $D \equiv D$

so (∇IND), $D \to_x E$, n so $D \to_x \to_c F$ −

- $D \to_x E$, n so $D \to_x \to_c F$ −

(IND) s s r −

(VIND) s s r − □

. − *For closed D if x is tagged in D and* $D \to_c^* E$ *then* $D \to_x^* \to_{\neg x}^* E$.

F − t $D \to_c^n E$, n pro n u t on on n −

- $n =$ t n $D \equiv E$ so $D \to_x^* \to_{\neg x}^* E$ −
- $n >$ t n $D \to_c F \to_c^{n-} E$, n ropos t on .x s t n F so n u t on $F \to_x^* \to_{\neg x}^* E$, so ropos t on $D \to_x^* \to_c \to_{\neg x}^* E$, n so $D \to_x^* \to_{\neg x}^* E$ − □

. − *For closed D if x is tagged in D*

$$
\begin{aligned}
&{}_{\gamma}\ \nu\vec{x}\,.\,(I, \nu(\mathsf{wv}\,G)\,.\,G) && \gamma\\
&\equiv\ \nu\vec{x}\,.\,H && \text{Eqn}\\
&\equiv\ E && \text{Eqn}
\end{aligned}
$$

us $D \rightarrow_x \quad {}_{\gamma} E$ − □

, \ − *For closed D if $D \rightarrow E$ then $D\Downarrow_x$ iff $E\Downarrow_x$*

F −

$\Rightarrow$ $D \rightarrow_c E$, t n ropos t on $E\Downarrow_x$ −

$D \rightarrow_\gamma E$, t n ropos t ons , \ n \

$$
\begin{array}{l}
\mathsf{tag}_x D \rightarrow_x \cdots \rightarrow_x F\\
\downarrow^{\le}_{\gamma}\\
\mathsf{tag}_x E
\end{array}
$$

- t r s $F_i = (x_i = M_i)$ n $w_i = \varepsilon$ –
  n
- For i su t t $D[\vec{x}/\vec{z}] \vdash x \sim x_i$ $(x_i = M_i[\vec{x}/\vec{z}]) \rightarrow_c \nu \vec{w}_i . F_i[\vec{x}/\vec{z}]$ n so $E \rightarrow_c^* F[\vec{x}/\vec{z}]$ –
- r $D[\vec{y}/\vec{z}] \rightarrow_c^* F[\vec{y}/\vec{z}]$ –
- t $\mathcal{R}$ ν ss $D[\vec{x}/\vec{z}]$ s u t on su t t $\vec{x}\ \mathcal{R}\ \vec{y}$ – n t $\mathcal{R}'$ t s st r t on ont n n $\mathcal{R}$ su t t $\vec{w}\vec{w}\vec{w}_i\vec{w}_i\ \mathcal{R}\ \vec{w}\vec{w}_j\vec{w}_j\vec{w}$ – n s o $\mathcal{R}'$ s ν ss $(G, x = M, F, \ldots, F_n)[\vec{x}/\vec{z}]$ s u t on n so $F[\vec{x}/\vec{z}] \vdash \vec{x} \sim \vec{y}$ – □

n (νMIG)

$$\nu\vec{x}\,.\,(D, \mathsf{local}\, G \;\mathsf{in}\; x \;= {}_{7}M', \mathsf{local}\, H \;\mathsf{in}\; y \;= {}_{7}N')$$

$$\equiv \nu\vec{x}\,.\,\nu(\mathsf{wv}\, G)\,.\,\nu(\mathsf{wv}\, H)\,.\,(D, G, H, x \;= {}_{7}M', y \;= {}_{7}N')$$

n ro t n t on s u t on

$$\nu\vec{x}\,.\,\nu(\mathsf{wv}\, G)\,.\,\nu(\mathsf{wv}\, H)\,.\,(D, G, H, x \;= {}_{7}M', y \;= {}_{7}N') \vdash x \sim y$$

so n u t on

$$\nu\vec{x}\,.\,\nu(\mathsf{wv}\, G)\,.\,\nu(\mathsf{wv}\, H)\,.\,(D, G, H, x \;= {}_{7}\nabla y, y \;= {}_{7}N')\Downarrow_z$$

n so

$$\begin{array}{llr}
& \nu\vec{x}\,.\,(D, x \;= {}_{7}\nabla y, y \;= {}_{7}N) & \\
\equiv & \nu\vec{x}\,.\,(D, x \;= {}_{7}\nabla y, y \;= {}_{7}\mathsf{rec}\, H \;\mathsf{in}\; N') & \text{Eqn} \\
\to & \nu\vec{x}\,.\,(D, x \;= {}_{7}\nabla y, \mathsf{local}\, H \;\mathsf{in}\; y \;= {}_{7}N') & \text{B D} \\
& \nu\vec{x}\,.\,(D, \mathsf{local}\, G \;\mathsf{in}\; \varepsilon, x \;= {}_{7}\nabla y, \mathsf{local}\, H \;\mathsf{in}\; y \;= {}_{7}N') & \gamma \\
\equiv & \nu\vec{x}\,.\,\nu(\mathsf{wv}\, G)\,.\,\nu(\mathsf{wv}\, H)\,.\,(D, G, H, x \;= {}_{7}\nabla y, y \;= {}_{7}N') & \nu\text{MIG}
\end{array}$$

n so Equ t on n ropos t on

$$\nu\vec{x}\,.\,(D, x \;= {}_{7}\nabla y, y \;= {}_{7}N)\Downarrow_z$$

ot r s s r s r

⇐ sy
wvi0 238.2.240113 12(m)-8302154(i)-5.01912(l)-5f 0 3 23364 11 -1 0 3.1126!

$$\perp \circ f = \perp$$

n so un or t

$$\mathsf{fix}(\mathsf{set}\, X g) \circ f = \mathsf{fix}(\mathsf{set}\, X g)$$

Fro t s t s s to s o n u t on on D t t $[\![D]\!] = [\![D]\!] \circ f$ –

–$(\mathsf{wv}[\![D]\!] \subseteq \mathsf{wv}\, D)$ An n u t on on D –

$(\mathsf{wv}[\![D]\!] \supseteq \mathsf{wv}\, D)$ $\mathsf{wv}[\![D]\!]$ $\mathsf{wv}\, D$ t n n $x \in \mathsf{wv}\, D$ n $x \notin \mathsf{wv}[\![D]\!]$ – n

$$\top$$

$$= \mathsf{read}\, x \circ (x \;= \top) \qquad \text{ropn}$$

$$= \mathsf{read}\, x \circ [\![D]\!] \circ (x \;= \top) \qquad x \notin \mathsf{wv}[\![D]\!]$$

$$= \mathsf{read}\, x \circ [\![$$

$= \mathsf{read}\, x \circ f$ $\quad f = g \circ f$

$x \notin X$ t n

$$\begin{aligned}
&\mathsf{read}\, x \circ (\mathsf{set}\, X g)^{n+} \perp \circ f \\
&= \mathsf{read}\, x \circ (\mathsf{set}\, X g)((\mathsf{set}\, X g)^{n} \perp) \circ f && \text{D n } f^{n} \\
&= \mathsf{read}\, x \circ f && \text{ropn}
\end{aligned}$$

us $(\mathsf{set}\, X g)^{n+} \perp \circ f \leq f$

us

$$\begin{aligned}
&f = g \circ f \\
&\Rightarrow \bigvee\{(\mathsf{set}\, X g)^{n} \perp \circ f \mid n \;\mathsf{in}\; \omega\} \leq f && \text{A o} \\
&\Rightarrow \bigvee\{(\mathsf{set}\, X g)^{n} \perp \mid n \;\mathsf{in}\; \omega\} \circ f \leq f && \circ \text{ s ont nuous} \\
&\Rightarrow \mathsf{fix}(\mathsf{set}\, X g) \circ f \leq f && \text{D n fix}
\end{aligned}$$

For p wv $f = X$ wv $g = Y$ n $X \cap Y = \emptyset$ t n v p rt

$$\mathsf{fix}(\mathsf{set}(X \cup Y)(f \circ g)) = f \circ \mathsf{fix}(\mathsf{set}(X \cup Y)(f \circ g))$$

n so t o

$$\mathsf{fix}(\mathsf{set}\, X f) \circ \mathsf{fix}(\mathsf{set}(X \cup Y)(f \circ g)) \leq \mathsf{fix}(\mathsf{set}(X \cup Y)(f \circ g))$$

r

$$\mathsf{fix}(\mathsf{set}\, Y g) \circ \mathsf{fix}(\mathsf{set}(X \cup Y)(f \circ g)) \leq \mathsf{fix}(\mathsf{set}(X \cup Y)(f \circ g))$$

us

$$\begin{aligned}
&\mathsf{set}(X \cup Y)(\mathsf{fix}(\mathsf{set}\, X f) \circ \mathsf{fix}(\mathsf{set}\, Y g))(\mathsf{fix}(\mathsf{set}(X \cup Y)(f \circ g))) \\
&= \mathsf{fix}(\mathsf{set}\, X f) \circ \mathsf{fix}(\mathsf{set}\, Y g) \circ \mathsf{fix}(\mathsf{set}(X \cup Y)(f \circ g)) && \text{ropn} \\
&\leq \mathsf{fix}(\mathsf{set}\, X f) \circ \mathsf{fix}(\mathsf{set}(X \cup Y)(f \circ g)) && \text{Eqn} \\
&\leq \mathsf{fix}(\mathsf{set}(X \cup Y)(f \circ g)) && \text{Eqn}
\end{aligned}$$

☛ $x \in \mathsf{wv}\, D$ t n

$[\![(\mathsf{rec}\, D\ \mathsf{in}\ M$

$$= [\![D]\!]$$

¬Assu [redacted]

[redacted] $= \neg M) \quad (w \quad \psi \to \chi)$

[redacted] $)\Downarrow_w$ so ropos t on ¬

$$(D, w = \neg M, x = \neg M)\Downarrow_x$$

[redacted] n $(z = x \quad y) \sqsubseteq E \sqsupseteq (D, w = \neg M, x = \neg M)$ t r

[redacted] $= w$ or $z = x$, so $M = x \quad y$, so $(D, w = \neg M, x = \neg M)\Uparrow_x$ s on

[redacted] s t on n n F su t t

$(F, w$ [redacted] $) = E$ $(F$

F –

–An n u t on on ϕ – on u t s s n $\phi = \psi \rightarrow \chi$ –

$\Rightarrow$ $\models D$ $(x$ $\psi \rightarrow \chi)$ t n $D\Downarrow_x$ so ropos t on $\nu w.\, D\Downarrow_x$ –For n $(z = x$ $y) \sqsubseteq E \sqsupseteq (\nu w.\, D)$ t v r s ropos t on n n $F \sqsupseteq (z = x$ $y)$ su t t

$$E \equiv \nu v.\, F \qquad F \sqsupseteq [v/w]D[v/w]$$

so ropos t on

$$\models [v/w]D[v/w] \quad (x \quad \psi \rightarrow \chi)$$

n so

$$\models E \quad (y \quad \psi)$$
$$\Rightarrow \models \nu v.\, F \quad (y \quad \psi)$$

• $w = x$ t n n n r s $\vec{y}$ n I su t t

$$H \equiv \nu\vec{x}\vec{y}\,.\,(F, G, I, w = M, z = w \quad y)$$

so t $\vec{w} = w v G$ n t v n $\vec{v}$ r s n s n $\models D \quad (x \quad \psi \to \chi)$ ropos t on

$$\nu\vec{x}\,.\,(F, v = \mathsf{rec}\, G \,\mathsf{in}\, M)[v/w] \quad (v \quad \psi \to \chi)$$

n ro t n t on $\sqsubseteq$

$$\begin{aligned}(z = v \quad y) &\\ \sqsubseteq\ &\nu\vec{x}\,.\,(F[v/w], G, I, v = (\mathsf{rec}\, G \,\mathsf{in}\, M)[v/w],\\ &w = M[v/w], z = v \quad y)\\ \sqsupseteq\ &\nu\vec{x}\,.\,(F, v = \mathsf{rec}\, G \,\mathsf{in}\, M)[v/w]\end{aligned}$$

n

$$\begin{aligned}&\models H \quad (y \quad \psi)\\ \Rightarrow\ &\models \nu\vec{x}\vec{y}\,.\,(F, G, I, w = M, z = w \quad y) \quad (y \quad \psi) && \text{Eqn}\\ \Rightarrow\ &\models (F, G, I, w = M, z = w \quad y) \quad (y \quad \psi) && \text{ropn}\\ \Rightarrow\ &\models (F, G, I, [v\vec{v}/w\vec{w}]G[v\vec{v}/w\vec{w}],\\ &\quad v = M, w = M, z = w \quad y) \quad (y \quad \psi) && \text{ropn}\\ \Rightarrow\ &\models (F[v/w], G, I, [v\vec{v}/w\vec{w}]G[v\vec{v}/w\vec{w}],\\ &\quad v = M[v/w], w = M[v/w], z = v \quad y) \quad (y \quad \psi) && \text{ropn}\\ \Rightarrow\ &\models (F[v/w], G, I, v = (\mathsf{rec}\, G \,\mathsf{in}\, M)[v/w],\\ &\quad w = M[v/w], z = v \quad y) \quad (y \quad \psi) && \text{n n}\\ \Rightarrow\ &\models (F[v/w], G, I, v = (\mathsf{rec}\, G \,\mathsf{in}\, M)[v/w],\\ &\quad w = M[v/w], z = v \quad y) \quad (z \quad \chi) && \text{Eqns n}\\ \Rightarrow\ &\models H \quad (z \quad \chi) && \text{r}\end{aligned}$$

us $\models E \quad (x \quad \psi \to \chi)$

• $x \neq w \neq z$ t n t pro s s r

(OTHER) $D \to_c E$ s pro t out B D t n n s o t t

$$D \sqsubseteq D' \quad \text{p} \quad \text{s } D' \to_c E' \sqsupseteq E$$

$$E \sqsubseteq E' \quad \text{p} \quad \text{s } D \sqsubseteq D' \to_c E'$$

n $\models D \quad (x \quad \psi \to \chi)$ t n $D \Downarrow_x$ so ropos t on $E \Downarrow_x$ n or n $(z = x \quad y) \sqsubseteq F \sqsupseteq E$ n n G su t t

$$F \equiv (G, z = x \quad y)$$

n t w r s so

$$(w = x \quad y) \sqsubseteq (G, w = x \quad y, z = x \quad y) \sqsupseteq E$$

n n $H \sqsupseteq D$ su t t

$$H \to_c F$$

n

$$\begin{aligned}&\models F \quad (y \quad \psi)\\ \Rightarrow\ &\models (G, z = x \quad y) \quad (y \quad \psi) && \text{Eqn}\\ \Rightarrow\ &\models (G, z = x \quad y, w = x \quad y) \quad (y \quad \psi) && \text{ropn}\\ \Rightarrow\ &\models (H, w = x \quad y) \quad (y \quad \psi) && \text{n n}\\ \Rightarrow\ &\models (H, w = x \quad y) \quad (w \quad \chi) && \models D \quad (x \quad \psi \to \chi)\\ \Rightarrow\ &\models (G, z = x \quad y, w = x \quad y) \quad (w \quad \chi) && \text{n n}\\ \Rightarrow\ &\models (G, z = x \quad y, w = x \quad y) \quad (z \quad \chi) && \text{ropn}\\ \Rightarrow\ &\models (G, z = x \quad y) \quad (z \quad \chi) && \text{ropn}\\ \Rightarrow\ &\models F \quad (z \quad \chi) && \text{Eqn}\end{aligned}$$

us or n $(z = x \quad y) \sqsubseteq F \sqsupseteq E$

$$\models F \quad (y \quad \psi) \Rightarrow \models F \quad (z \quad \chi)$$

so $\models E \quad (x \quad \psi \to \chi)$

ot r r t on s s o n s r □

3.11 Full abstraction

n t s s t on s o t t t o **D** s u str t or on urr nt p r u t on s nst t on urr nt p r u t on s t s u str t o s t ost out r ostr u t on n so on urr nt p r u t on s t t s o put t on po r s t ost out r ostr u t on s pro o o s t s stru tur s t on

• s o t t $\Gamma \vdash D \quad \Delta$ $[\![\Delta]\!] \leq [\![D]\!][\![\Gamma]\!]$ t us s o t t t pro s st s soun n o p t or t not t on s nt s s s ropos t on t p r u t on qu nt ropos t on

• t n s o t t $\Gamma \vdash D \quad \Delta$ t n $\Gamma \models D \quad \Delta$ n t t $\Gamma \models D \quad \Delta$ t n $[\![\Delta]\!] \leq [\![D]\!][\![\Gamma]\!]$ us t t r pr s nt t ons t o r qu nt s s ropos t on t p r u t on qu nt ropos t on

• F n s o t t u st t on s n pro n t t r o pr s nt t ons to qu nt s s ropos t on t p r u t on qu nt ropos t on

us AB A Y n G s t n qu s n pt to p r u t on

$\Gamma \vdash M \quad \phi$ *iff* $[\![\phi]\!] \leq [\![M]\!][\![\Gamma]\!]$

$\Gamma \vdash D \quad \Delta$ *iff* $[\![\Delta]\!] \leq [\![D]\!][\![\Gamma]\!]$

F

D E $\Rightarrow$ v to prov t ru s $\Gamma \vdash M$ ϕ n $\Gamma \vdash D$ Δ soun For p to prov ( ) $[\![\Delta]\!] \leq [\![x = M]\!][\![\Gamma]\!]$ n $[\![\phi]\!] \leq [\![M]\!][\![\Delta]\!]$ t n

$$
\begin{aligned}
& [\![x \ \ \phi]\!] \\
& \leq (x = [\![M]\!])[\![\Delta]\!] && \text{H pot s s} \\
& \leq (x = [\![M]\!])([\![x = M]\!][\![\Gamma]\!]) && \text{H pot s s} \\
& = [\![x = M]\!][\![\Gamma]\!] && \text{ropn}
\end{aligned}
$$

ot r s s r s r

C E E E $\Leftarrow$ An n u t on on M n D For p $x \neq y$ n

$$[\![\phi]\!] \leq [\![x \ \ y]\!][\![\Gamma]\!]$$

t n t r $[\![\phi]\!] = \bot$ so $\vdash \phi = \omega$ n so $\Gamma \vdash x \ \ y \ \ \phi$ or

$$
\begin{aligned}
& [\![\phi]\!] \leq [\![x \ \ y]\!][\![\Gamma]\!] \\
& \Rightarrow [\![\phi]\!] \leq \mathsf{apply}[\![\Gamma(x)]\!][\![\Gamma(y)]\!] && \text{D n } [\![x \ \ y]\!] \\
& \Rightarrow [\![\Gamma(y) \rightarrow \phi]\!] \leq [\![\Gamma(x)]\!] && \text{ropn} \\
& \Rightarrow \vdash \Gamma(x) \leq \Gamma(y) \rightarrow \phi && \text{ropn} \\
& \Rightarrow \vdash \Gamma \leq x \ \ \Gamma(y) \rightarrow \phi, y \ \ \Gamma(y) && \text{D n } \leq \\
& \Rightarrow \Gamma \vdash x \ \ y \ \ \phi && (\leq)
\end{aligned}
$$

$$(z \mapsto x\;\; y) \sqsubseteq E \sqsupseteq (D, x \mapsto \lambda w.\, M)$$

- $\models D \quad (x \quad \phi \rightarrow \psi)$ t n $D\Downarrow_x$ so Coro r $[\![D]\!]\sigma x \neq \bot$ ¬A so or
r s y n z

true

$\Rightarrow$

4 Conclusions

nt sp p r v n v st t t r t ons p t nt s nt not on *full abstraction* n t p nt t on t n qu *concurrent graph reduction* – v s o n t t

- Con urr nt r p r u t on n v n s p op r t on pr s nt t on n t st BE Y n B D s *chemical abstract machine* n E s *polyadic π calculus* –
- t n qu s AB A Y n G s *lazy λ calculus* n

urs v r t ons o v r s or s AD H so n v st t st r t ons p t n r p r u t on n t $\mathbf{D}_\infty$ o t unt p λ u us s BA E D EG or or t s top s t r p up E E n ABA Y n G

BA A D EG *et al.* r s r o or on *term graph rewriting* ntro u BA A D EG *et al.* n sur v E A AY *et al.* n t ot r p p rs n EE *et al.* s EE *et al.* oo r r p s r v r s r to r t ons ut r root n oss B A

H HA A A D EA A — Anot r ppro to t op r t on s nt s r p r u t on s H HA A n EA A s AZY CF HA t n s s CF t let r t ons — s s v n st p op r t on s nt s t or n our s nt

$$(\mathsf{let}\,D\,\mathsf{in}\,M) \Downarrow (\mathsf{let}\,E\,\mathsf{in}\,N)$$

s s nt s s s r to ours n A CHB Y s pt t t

- AZY CF HA s t p n u n s onstru tors n onstru tors or oo ns n n tur nu rs —
- n let pr ss ons r n us r t r t n rec pr ss ons t s n t s or po nts os so s r n n or t on

$$(\mathsf{let}\,D\,\mathsf{in}\,\mathsf{let}\,x \;=_7 (\mu x\,.\,M)\;\mathsf{in}\,M) \Downarrow (\mathsf{let}\,E\,\mathsf{in}\,N)$$

F n n proo t n qu t t s po u nou to s o u str t on or on urr nt p r u t on ut o s not r on on s n s s s to qu t u t

Y ED λ CA C proo s n n t s p p r r on or t unt p λ u us t r urs v r t ons non str t un t on n u s r us n pr t r t p n v t p onstru tors n onstru tors usu n t or o p tt rn t n

u onstru tors n onstru tors ou to t λ u us t r urs v r t ons For p t pro u t t p $T \times U$ t onstru tors n onstru tors

$$\mathsf{pair} \quad T \to U \to (T \times U) \qquad \mathsf{fst} \quad (T \times U) \to T \qquad \mathsf{snd} \quad (T \times U) \to U$$

ou to t λ u us t r urs v r t ons s

$$M \quad = \cdots \mid \mathsf{pair}\, x y \mid \mathsf{fst}\, x \mid \mathsf{snd}\, x$$

t t op r t on s nt s or

⊥, l or r — choose unction ou to t λ u us t r urs v r t ons s

$$M = \cdots \mid \mathsf{choose}\, x y$$
$$D = \cdots \mid \mathsf{o} = _7 \Vert \mid \mathsf{o} = _7 \mathsf{l} \mid \mathsf{o} = _7 \mathsf{r}$$

t t op r t on s nt s v n

$$x = _7 \mathsf{choose}\, yz, y = ?M \mapsto x = _7 \mathsf{choose}\, yz, y = _7 M$$
$$x = _7 \mathsf{choose}\, yz, z = ?M \mapsto x = _7 \mathsf{choose}\, yz, z = _7 M$$
$$\mathsf{o} = _7 \Vert, x = _7 \mathsf{choose}\, yz, y = _7 \lambda w.\, M \mapsto \mathsf{o} = _7 \mathsf{l}, x = _7 \mathsf{choose}\, yz, y = _7 \lambda w.\, M$$
$$\mathsf{o} = _7 \Vert, x = _7 \mathsf{choose}\, yz, z = _7 \lambda w.\, M$$

E B D *Combinator Graph Reduction A Congruence and its Applications* D t s or n rs t

AC A E *Categories for the orking Mathematician* Gr u t ts n t t s pr n r r

B Fu str t s nt s t p λ u *Theoret. Comput. Sci.*

B *Communication and Concurrency* r nt H

B po π u us tutor n *Proc. International Summer School on Logic and Algebra of Specification* r to r o

H u us o s pro r n n s D ss rt t on

YO F A *Abstract Interpretation and Optimising Transformations for Applicative Programs* D t s E n ur n rs t D pt Co put r n

G C H *The Lazy Lambda Calculus An Investigation into the Foundations of Functional Programming* D t s p r Co on on n rs t

G C H on t r n s n un t on s tt n n *Proc. LICS* p s EEE Co put r o r ss

EY E *The Implementation of Functional Programming Languages* r nt H

B CE B C *Basic Category Theory for Computer Scientists* pr ss

G CF ons r s pro r n n *Theoret. Comput. Sci.*

G Do ns non ous tp

H HA A n EA A An qu t op r t on s nt s s r n n u t on n *Proc. ESOP*

E H E p t su st tut on nt s not D D n rs t Cop n n

C D Do ns or not t on s nt s n E E n CH D E tors *Proc. ICALP* p s pr n r r C

EE A E B n A EE E E C D tors *Term Graph Rewriting Theory and Practice* on n ons

E *Denotational Semantics The Scott Strachey Approach to Programming Language Theory* r ss

B D A n p nt t on t n qu or pp t n s *Software Practice and Experience*

B D r n A non str t un t on n t po orp t p s n *Proc. IFIP Conf. Functional Programming Languages and Computer Architecture* pr n r r C

AD H C *Semantics and Pragmatics of the Lambda Calculus* D t s or n rs t

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E
s u t on
s u t on v ss
s t or
split